that is,

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1. $R\cos\delta = 3$ and $R\sin\delta = 4$, so $R = \sqrt{25} = 5$ and $\delta = \arctan(4/3)$. We obtain that $u = 5\cos(2t - \arctan(4/3))$.

3. $R\cos\delta = 4$ and $R\sin\delta = -2$, so $R = \sqrt{20} = 2\sqrt{5}$ and $\delta = -\arctan(1/2)$. We obtain that $u = 2\sqrt{5}\,\cos(3t + \arctan(1/2))$.

4. $R\cos\delta = -2$ and $R\sin\delta = -3$, so $R = \sqrt{13}$ and $\delta = \pi + \arctan(3/2)$. We obtain that $u = \sqrt{13}\cos(\pi t - \pi - \arctan(3/2))$.

5. The spring constant is $k = 2/(1/2) = 4$ lb/ft. Mass $m = 2/32 = 1/16$ lb-s^2/ft. Since there is no damping, the equation of motion is $u''/16 + 4u = 0$, that is, $u'' + 64u = 0$. The initial conditions are $u(0) = 1/4$ ft, $u'(0) = 0$ ft/s. The general solution is $u(t) = A\cos 8t + B\sin 8t$. Invoking the initial conditions, we have $u(t) = \cos 8t/4$. $R = 1/4$ ft, $\delta = 0$ rad, $\omega_0 = 8$ rad/s, and $T = \pi/4$ s.

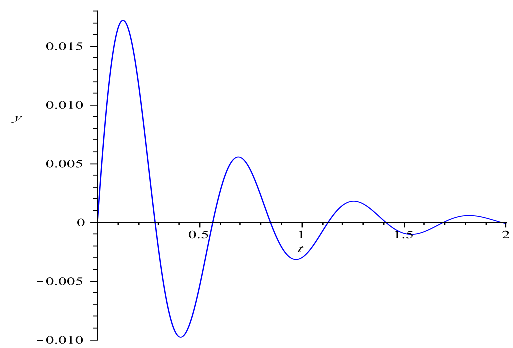
7. The spring constant is $k = 3/(1/4) = 12$ lb/ft. Mass $m = 3/32$ lb-s^2/ft. Since there is no damping, the equation of motion is $3u''/32 + 12u = 0$, that is, $u'' + 128u = 0$. The initial conditions are $u(0) = -1/12$ ft, $u'(0) = 2$ ft/s. The general solution is $u(t) = A\cos 8\sqrt{2}\,t + B\sin 8\sqrt{2}\,t$. Invoking the initial conditions, we have

$$u(t) = -\frac{1}{12}\cos 8\sqrt{2}\,t + \frac{1}{4\sqrt{2}}\sin 8\sqrt{2}\,t\,.$$

$R = \sqrt{11/288}$ ft, $\delta = \pi - \arctan(3/\sqrt{2})$ rad, $\omega_0 = 8\sqrt{2}$ rad/s, $T = \pi/(4\sqrt{2})$ s.

10. The spring constant is $k = 16/(1/4) = 64$ lb/ft. Mass $m = 1/2$ lb-s^2/ft. The damping coefficient is $\gamma = 2$ lb-s/ft. Hence the equation of motion is $u''/2 + 2u' + 64u = 0$, that is, $u'' + 4u' + 128u = 0$. The initial conditions are $u(0) = 0$ ft, $u'(0) = 1/4$ ft/s. The general solution is $u(t) = A \cos 2\sqrt{31}\,t + B \sin 2\sqrt{31}\,t$. Invoking the initial conditions, we have

$$u(t) = \frac{1}{8\sqrt{31}} e^{-2t} \sin 2\sqrt{31}\,t\,.$$



Solving $u(t) = 0$, on the interval $[0.2\,,\,0.4]$, we obtain $t = \pi/2\sqrt{31} = 0.2821$ s. Based on the graph, and the solution of $u(t) = -0.01/12$ ft, we have $|u(t)| \le 0.01$ for $t \ge \tau = 1.5927$.

11. The spring constant is $k = 3/(.1) = 30$ N/m. The damping coefficient is given as $\gamma = 3/5$ N-s/m. Hence the equation of motion is $2u'' + 3u'/5 + 30u = 0$, that is, $u'' + 0.3u' + 15u = 0$. The initial conditions are $u(0) = 0.05$ m and $u'(0) = 0.01$ m/s. The general solution is $u(t) = A \cos \mu t + B \sin \mu t$, in which $\mu = 3.87008$ rad/s. Invoking the initial conditions, we have $u(t) = e^{-0.15t}(0.05 \cos \mu t + 0.00452 \sin \mu t)$. Also, $\mu/\omega_0 = 3.87008/\sqrt{15} \approx 0.99925$.

13. The frequency of the undamped motion is $\omega_0 = 1$. The quasi frequency of the damped motion is $\mu = \sqrt{4 - \gamma^2}\,/2$. Setting $\mu = 2\omega_0/3$, we obtain $\gamma = 2\sqrt{5}\,/3$.

14. The spring constant is $k = mg/L$. The equation of motion for an undamped system is $mu'' + mgu/L = 0$. Hence the natural frequency of the system is $\omega_0 = \sqrt{g/L}$. The period is $T = 2\pi/\omega_0$.

15. The general solution of the system is $u(t) = A \cos \gamma(t - t_0) + B \sin \gamma(t - t_0)$. Invoking the initial conditions, we have $u(t) = u_0 \cos \gamma(t - t_0) + (u_0'/\gamma) \sin \gamma(t - t_0)$. Clearly, the functions $v = u_0 \cos \gamma(t - t_0)$ and $w = (u_0'/\gamma) \sin \gamma(t - t_0)$ satisfy the given criteria.

16. Note that $r \sin(\omega_0 t - \theta) = r \sin \omega_0 t \cos \theta - r \cos \omega_0 t \sin \theta$. Comparing the given expressions, we have $A = -r \sin \theta$ and $B = r \cos \theta$. That is, $r = R = \sqrt{A^2 + B^2}$, and $\tan \theta = -A/B = -1/\tan \delta$. The latter relation is also $\tan \theta + \cot \delta = 1$.

18. The system is critically damped, when $R = 2\sqrt{L/C}$. Here $R = 1000$ ohms.

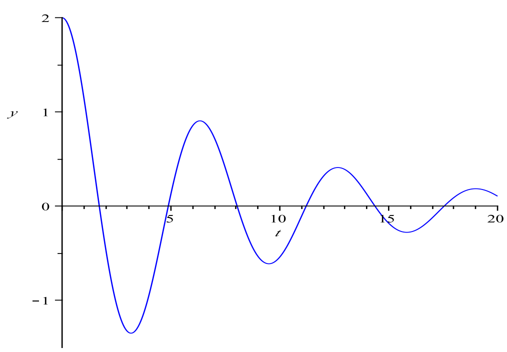
21.(a) Let $u = Re^{-\gamma t/2m}\cos(\mu t - \delta)$. Then attains a maximum when $\mu t_k - \delta = 2k\pi$. Hence $T_d = t_{k+1} - t_k = 2\pi/\mu$.

(b) $u(t_k)/u(t_{k+1}) = e^{-\gamma t_k/2m}/e^{-\gamma t_{k+1}/2m} = e^{(\gamma t_{k+1} - \gamma t_k)/2m}$. Hence $u(t_k)/u(t_{k+1}) = e^{\gamma(2\pi/\mu)/2m} = e^{\gamma T_d/2m}$.

(c) $\Delta = \ln\left[u(t_k)/u(t_{k+1})\right] = \gamma(2\pi/\mu)/2m = \pi\gamma/\mu m$.

22. The spring constant is $k = 16/(1/4) = 64$ lb/ft. Mass $m = 1/2$ lb-s^2/ft. The damping coefficient is $\gamma = 2$ lb-s/ft. The quasi frequency is $\mu = 2\sqrt{31}$ rad/s. Hence $\Delta = 2\pi/\sqrt{31} \approx 1.1285$.

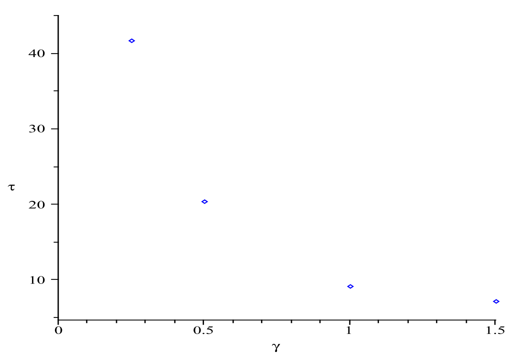
25.(a) The solution of the IVP is $u(t) = e^{-t/8}(2\cos 3\sqrt{7}\,t/8 + (2\sqrt{7}/21)\sin 3\sqrt{7}\,t/8)$.



Using the plot, and numerical analysis, $\tau \approx 41.715$.

(b) For $\gamma = 0.5$, $\tau \approx 20.402$; for $\gamma = 1.0$, $\tau \approx 9.168$; for $\gamma = 1.5$, $\tau \approx 7.184$.

(c)



(d) For $\gamma = 1.6$, $\tau \approx 7.218$; for $\gamma = 1.7$, $\tau \approx 6.767$; for $\gamma = 1.8$, $\tau \approx 5.473$; for $\gamma = 1.9$, $\tau \approx 6.460$. τ steadily decreases to about $\tau_{min} \approx 4.873$, corresponding to the critical value $\gamma_0 \approx 1.73$.

(e) We can rewrite the solution as $u(t) = Re^{-\gamma t/2}\cos(\mu t - \delta)$, where $R = 4/\sqrt{4 - \gamma^2}$.

Neglecting the cosine factor, we can approximate τ by solving $Re^{-\gamma\tau/2} = 1/100$, thus finding

$$\tau \approx \frac{2}{\gamma}\ln(100R) = \frac{2}{\gamma}\ln\left(\frac{400}{\sqrt{4-\gamma^2}}\right).$$

For $\gamma = 0.25$, $\tau \approx 42.4495$; for $\gamma = 0.5$, $\tau \approx 21.3223$; for $\gamma = 1.0$, $\tau \approx 10.8843$; for $\gamma = 1.5$, $\tau \approx 7.61554$; for $\gamma = 1.6$, $\tau \approx 7.26143$; for $\gamma = 1.7$, $\tau \approx 6.98739$; for $\gamma = 1.8$, $\tau \approx 6.80965$; for $\gamma = 1.9$, $\tau \approx 6.80239$.

26.(a) The characteristic equation is $mr^2 + \gamma r + k = 0$. Since $\gamma^2 < 4km$, the roots are $r_{1,2} = (-\gamma \pm i\sqrt{4mk-\gamma^2})/2m$. The general solution is

$$u(t) = e^{-\gamma t/2m}\left[A\,\cos\frac{\sqrt{4mk-\gamma^2}}{2m}t + B\,\sin\frac{\sqrt{4mk-\gamma^2}}{2m}t\right].$$

Invoking the initial conditions, $A = u_0$ and $B = (2mv_0 - \gamma u_0)/\sqrt{4mk-\gamma^2}$.

(b) We can write $u(t) = R\,e^{-\gamma t/2m}\cos(\mu t - \delta)$, in which

$$R = \sqrt{u_0^2 + \frac{(2mv_0-\gamma u_0)^2}{4mk-\gamma^2}} \quad \text{and} \quad \delta = \arctan\left[\frac{(2mv_0-\gamma u_0)}{u_0\sqrt{4mk-\gamma^2}}\right].$$

(c)

$$R = \sqrt{u_0^2 + \frac{(2mv_0-\gamma u_0)^2}{4mk-\gamma^2}} = 2\sqrt{\frac{m(ku_0^2 + \gamma u_0 v_0 + mv_0^2)}{4mk-\gamma^2}} = \sqrt{\frac{a+b\gamma}{4mk-\gamma^2}}.$$

It is evident that R increases (monotonically) without bound as $\gamma \to (2\sqrt{mk})^-$.

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