[redacted]

2. We have $\sin(\alpha \pm \beta) = \sin\,\alpha\,\cos\,\beta \pm \cos\,\alpha\,\sin\,\beta$. Subtracting the two identities, we obtain $\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\,\cos\,\alpha\,\sin\,\beta$. Setting $\alpha+\beta = 7t$ and $\alpha - \beta = 6t$, we get that $\alpha = 6.5t$ and $\beta = 0.5t$. This implies that $\sin\,7t - \sin\,6t = 2\,\sin\,(t/2)\cos\,(13t/2)$.

3. Consider the trigonometric identities $\cos(\alpha \pm \beta) = \cos\,\alpha\,\cos\,\beta \mp \sin\,\alpha\,\sin\,\beta$. Adding the two identities, we get $\cos(\alpha-\beta) + \cos(\alpha+\beta) = 2\,\cos\,\alpha\,\cos\,\beta$. Comparing the expressions, set $\alpha+\beta = 2\pi t$ and $\alpha - \beta = \pi t$. This means $\alpha = 3\pi t/2$ and $\beta = \pi t/2$. Upon substitution, we have $\cos(\pi t) + \cos(2\pi t) = 2\,\cos(3\pi t/2)\,\cos(\pi t/2)$.

4. Adding the two identities $\sin(\alpha \pm \beta) = \sin\,\alpha\,\cos\,\beta \pm \cos\,\alpha\,\sin\,\beta$, it follows that $\sin(\alpha-\beta) + \sin(\alpha+\beta) = 2\sin\,\alpha\,\cos\,\beta$. Setting $\alpha+\beta = 4t$ and $\alpha-\beta = 3t$, we have $\alpha = 7t/2$ and $\beta = t/2$. Hence $\sin\,3t + \sin\,4t = 2\,\sin(7t/2)\,\cos(t/2)$.

6. Using MKS units, the spring constant is $k = 5(9.8)/0.1 = 490$ N/m, and the damping coefficient is $\gamma = 2/0.04 = 50$ N-s/m. The equation of motion is

$$5u'' + 50u' + 490u = 10\,\sin(t/2)\,.$$

The initial conditions are $u(0) = 0$ m and $u'(0) = 0.03$ m/s.

8.(a) The homogeneous solution is $u_c(t) = Ae^{-5t}\cos\sqrt{73}\,t + Be^{-5t}\sin\sqrt{73}\,t$. Based on the method of undetermined coefficients, the particular solution is

$$U(t) = \frac{1}{153281}\left[-160\,\cos(t/2) + 3128\,\sin(t/2)\right].$$

Hence the general solution of the ODE is $u(t) = u_c(t) + U(t)$. Invoking the initial conditions, we find that

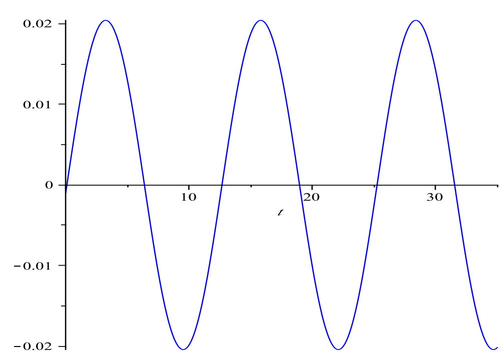
$$A = 160/153281 \text{ and } B = 383443\sqrt{73}\,/1118951300\,.$$

Hence the response is

$$u(t) = \frac{1}{153281}\left[160\,e^{-5t}\cos\,\sqrt{73}\,t + \frac{383443\sqrt{73}}{7300}e^{-5t}\sin\,\sqrt{73}\,t\right] + U(t).$$

(b) $u_c(t)$ is the transient part and $U(t)$ is the steady state part of the response.

(c)



(d) The amplitude of the forced response is given by $R = 2/\Delta$, in which

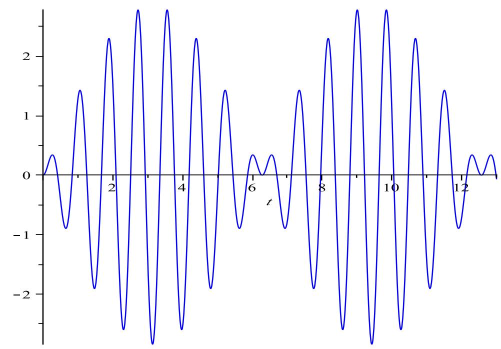
$$\Delta = \sqrt{25(98-\omega^2)^2 + 2500\,\omega^2}\,.$$

The maximum amplitude is attained when Δ is a minimum. Hence the amplitude is maximum at $\omega = 4\sqrt{3}$ rad/s.

9. The spring constant is $k = 12$ lb/ft and hence the equation of motion is

$$\frac{6}{32}u'' + 12u = 4\,\cos\,7t\,,$$

that is, $u'' + 64u = (64/3)\cos 7t$. The initial conditions are $u(0) = 0$ ft, $u'(0) = 0$ ft/s. The general solution is $u(t) = A\cos 8t + B\sin 8t + (64/45)\cos 7t$. Invoking the initial conditions, we have $u(t) = -(64/45)\cos 8t + (64/45)\cos 7t = (128/45)\sin(t/2)\sin(15t/2)$.



12. The equation of motion is $2u'' + u' + 3u = 3\cos 3t - 2\sin 3t$. Since the system is damped, the steady state response is equal to the particular solution. Using the method of undetermined coefficients, we obtain $u_{ss}(t) = (\sin 3t - \cos 3t)/6$. Further, we find that $R = \sqrt{2}/6$ and $\delta = \arctan(-1) = 3\pi/4$. Hence we can write $u_{ss}(t) = (\sqrt{2}/6)\cos(3t - 3\pi/4)$.

13.(a,b) Plug in $u(t) = R\cos(\omega t - \delta)$ into the equation $mu'' + \gamma u' + ku = F_0\cos\omega t$, then use trigonometric identities and compare the coefficients of $\cos\omega t$ and $\sin\omega t$. The result follows.

(c) The amplitude of the steady-state response is given by

$$R = \frac{F_0}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + \gamma^2\,\omega^2}}\,.$$

Since F_0 is constant, the amplitude is maximum when the denominator of R is minimum. Let $z = \omega^2$, and consider the function $f(z) = m^2(\omega_0^2 - z)^2 + \gamma^2 z$. Note that $f(z)$ is a quadratic, with minimum at $z = \omega_0^2 - \gamma^2/2m^2$. Hence the amplitude R attains a maximum at $\omega_{max}^2 = \omega_0^2 - \gamma^2/2m^2$. Furthermore, since $\omega_0^2 = k/m$,

$$\omega_{max}^2 = \omega_0^2\left[1 - \frac{\gamma^2}{2km}\right].$$

Substituting $\omega^2 = \omega_{max}^2$ into the expression for the amplitude,

$$R = \frac{F_0}{\sqrt{\gamma^4/4m^2 + \gamma^2\,(\omega_0^2 - \gamma^2/2m^2)}} = \frac{F_0}{\sqrt{\omega_0^2\gamma^2 - \gamma^4/4m^2}} = \frac{F_0}{\gamma\omega_0\sqrt{1 - \gamma^2/4mk}}\,.$$

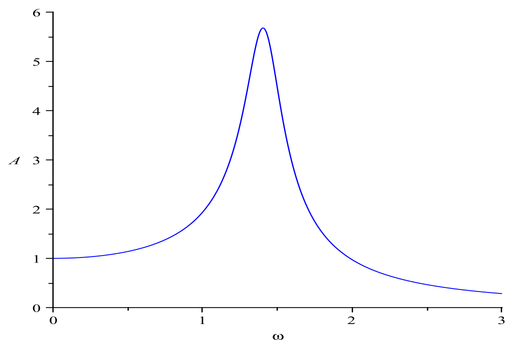
17.(a) The steady state part of the solution $U(t) = A\cos\omega t + B\sin\omega t$ may be found by substituting this expression into the differential equation and solving for A and B. We find that

$$A = \frac{32(2 - \omega^2)}{64 - 63\omega^2 + 16\omega^4}, \qquad B = \frac{8\omega}{64 - 63\omega^2 + 16\omega^4}.$$

(b) The amplitude is

$$A = \frac{8}{\sqrt{64 - 63\omega^2 + 16\,\omega^4}}\,.$$

(c)



(d) See Problem 13. The amplitude is maximum when the denominator of A is minimum. That is, when $\omega = \omega_{max} = 3\sqrt{14}\,/8 \approx 1.4031$. Hence $A = 64/\sqrt{127}$.

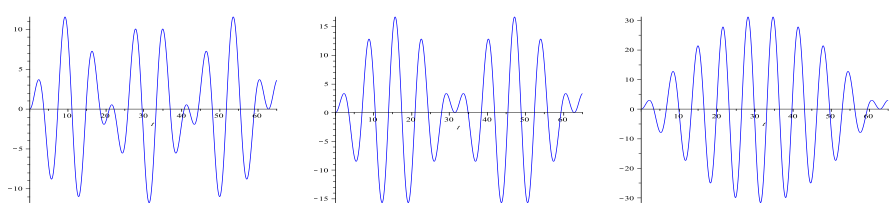
18.(a) The homogeneous solution is $u_c(t) = A\cos\, t + B\sin\, t$. Based on the method of undetermined coefficients, the particular solution is

$$U(t) = \frac{3}{1-\omega^2}\,\cos\,\omega t\,.$$

Hence the general solution of the ODE is $u(t) = u_c(t) + U(t)$. Invoking the initial conditions, we find that $A = 3/(\omega^2 - 1)$ and $B = 0$. Hence the response is

$$u(t) = \frac{3}{1-\omega^2}\,[\,\cos\,\omega t - \cos\, t\,]\,.$$

(b)



(a) $\omega = 0.7$ (b) $\omega = 0.8$ (c) $\omega = 0.9$

Note that

$$u(t) = \frac{6}{1-\omega^2}\,\sin\left[\frac{(1-\omega)t}{2}\right]\sin\left[\frac{(\omega+1)t}{2}\right].$$

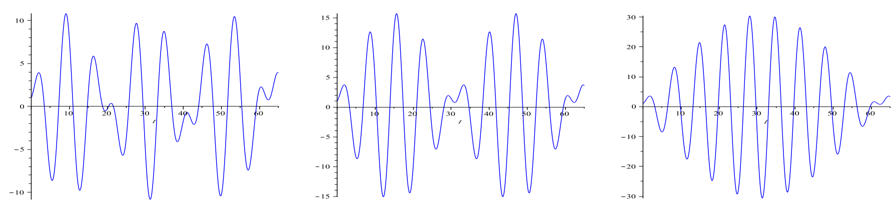
19.(a) The homogeneous solution is $u_c(t) = A\cos t + B\sin t$. Based on the method of undetermined coefficients, the particular solution is

$$U(t) = \frac{3}{1-\omega^2}\cos \omega t\,.$$

Hence the general solution is $u(t) = u_c(t) + U(t)$. Invoking the initial conditions, we find that $A = (\omega^2+2)/(\omega^2-1)$ and $B = 1$. Hence the response is

$$u(t) = \frac{1}{1-\omega^2}\left[3\cos \omega t - (\omega^2+2)\cos t\right] + \sin t\,.$$

(b)



(a) $\omega = 0.7$ (b) $\omega = 0.8$ (c) $\omega = 0.9$

Note that

$$u(t) = \frac{6}{1-\omega^2}\sin\left[\frac{(1-\omega)t}{2}\right]\sin\left[\frac{(\omega+1)t}{2}\right] + \cos t + \sin t\,.$$

[redacted]