[redacted]

1. Rewriting the equation as

$$y' + \frac{\ln t}{t-3} y = \frac{2t}{t-3}$$

and using Theorem 2.4.1, we conclude that a solution is guaranteed to exist in the interval $0 < t < 3$.

2. Rewriting the equation as

$$y' + \frac{1}{t(t-4)} y = 0$$

and using Theorem 2.4.1, we conclude that a solution is guaranteed to exist in the interval $0 < t < 4$.

3. By Theorem 2.4.1, we conclude that a solution is guaranteed to exist in the interval $\pi/2 < t < 3\pi/2$.

4. Rewriting the equation as

$$y' + \frac{2t}{4-t^2} y = \frac{3t^2}{4-t^2}$$

and using Theorem 2.4.1, we conclude that a solution is guaranteed to exist in the interval $-\infty < t < -2$.

5. Rewriting the equation as

$$y' + \frac{2t}{4-t^2} y = \frac{3t^2}{4-t^2}$$

and using Theorem 2.4.1, we conclude that a solution is guaranteed to exist in the interval $-2 < t < 2$.

6. Rewriting the equation as

$$y' + \frac{1}{lnt} y = \frac{\cot t}{lnt}$$

and using Theorem 2.4.1, we conclude that a solution is guaranteed to exist in the interval $1 < t < \pi$.

7. Using the fact that

$$f = \frac{t-y}{2t+5y} \implies f_y = \frac{3t-10y}{(2t+5y)^2},$$

we see that the hypothesis of Theorem 2.4.2 are satisfied as long as $2t + 5y \neq 0$.

8. Using the fact that

$$f = (1-t^2-y^2)^{1/2} \implies f_y = -\frac{y}{(1-t^2-y^2)^{1/2}},$$

we see that the hypothesis of Theorem 2.4.2 are satisfied as long as $t^2 + y^2 < 1$.

9. Using the fact that

$$f = \frac{ln|ty|}{1-t^2+y^2} \implies f_y = \frac{1-t^2+y^2-2y^2\ln|ty|}{y(1-t^2+y^2)^2},$$

we see that the hypothesis of Theorem 2.4.2 are satisfied as long as $y, t \neq 0$ and $1-t^2+y^2 \neq 0$.

10. Using the fact that

$$f = (t^2+y^2)^{3/2} \implies f_y = 3y(t^2+y^2)^{1/2},$$

we see that the hypothesis of Theorem 2.4.2 are satisfied for all $t \in \mathbb{R}$.

11. Using the fact that

$$f = \frac{1+t^2}{3y-y^2} \implies f_y = -\frac{(1+t^2)(3-2y)}{(3y-y^2)^2},$$

we see that the hypothesis of Theorem 2.4.2 are satisfied as long as $y \neq 0, 3$.

12. Using the fact that

$$f = \frac{(\cot t)y}{1+y} \implies f_y = \frac{1}{(1+y)^2},$$

we see that the hypothesis of Theorem 2.4.2 are satisfied as long as $y \neq -1, t \neq n\pi$ for $n = 0, 1, 2 \ldots$.

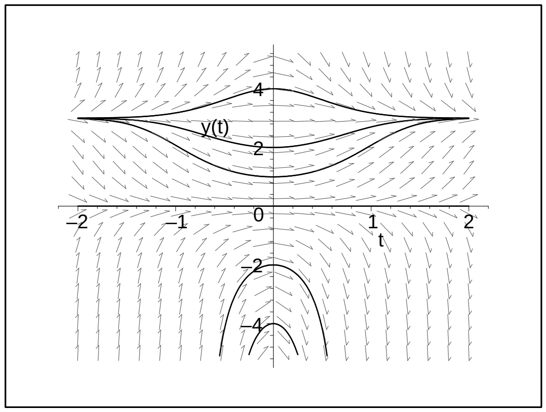
13. The equation is separable, $ydy = -4tdt$. Integrating both sides, we conclude that $y^2/2 = -2t^2 + y_0^2/2$ for $y_0 \neq 0$. The solution is defined for $y_0^2 - 4t^2 \geq 0$.

14. The equation is separable and can be written as $dy/y^2 = 2tdt$. Integrating both sides, we arrive at the solution $y = y_0/(1-y_0t^2)$. For $y_0 > 0$, solutions exist as long as $t^2 < 1/y_0$. For $y_0 \leq 0$, solutions exist for all t.

15. The equation is separable and can be written as $dy/y^3 = -dt$. Integrating both sides, we arrive at the solution $y = y_0/(\sqrt{2ty_0^2+1})$. Solutions exist as long as $2y_0^2t + 1 > 0$.

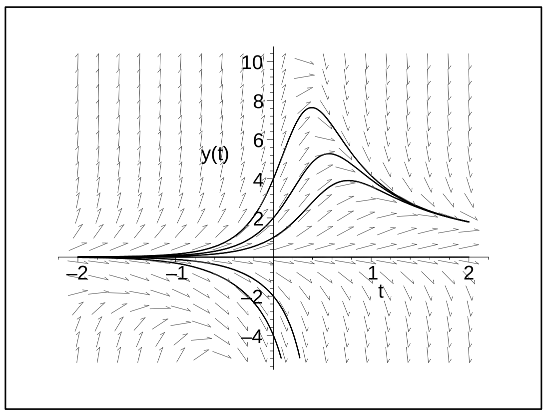
16. The equation is separable and can be written as $ydy = t^2dt/(1+t^3)$. Integrating both sides, we arrive at the solution $y = \pm(\frac{2}{3}\ln|1+t^3| + y_0^2)^{1/2}$. The sign of the solution depends on the sign of the initial data y_0. Solutions exist as long as $\frac{2}{3}\ln|1+t^3| + y_0^2 \geq 0$; that is, as long as $y_0^2 \geq -\frac{2}{3}\ln|1+t^3|$. We can rewrite this inequality as $|1+t^3| \geq e^{-3y_0^2/2}$. In order for the solution to exist, we need $t > -1$ (since the term $t^2/(1+t^3)$ has a singularity at $t = -1$. Therefore, we can conclude that our solution will exist for $[e^{-3y_0^2/2} - 1]^{1/3} < t < \infty$.

17.



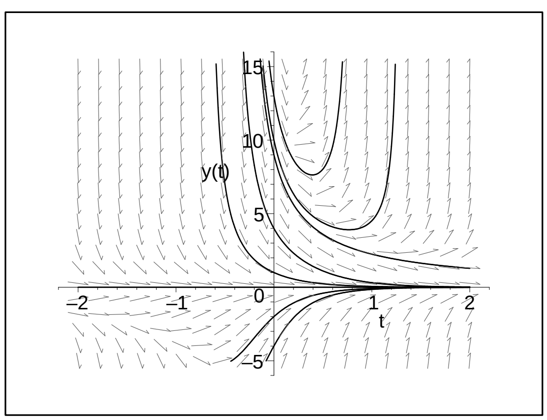
If $y_0 > 0$, then $y \to 3$. If $y_0 = 0$, then $y = 0$. If $y_0 < 0$, then $y \to -\infty$.

18.



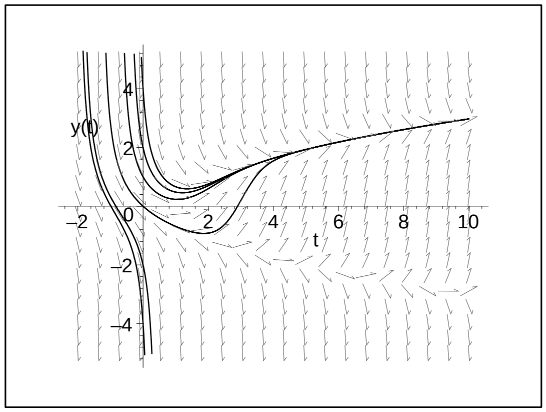
If $y_0 \geq 0$, then $y \to 0$. If $y_0 < 0$, then $y \to -\infty$.

19.



If $y_0 > 9$, then $y \to \infty$. If $y_0 < 9$, then $y \to 0$.

20.



If $y_0 < y_c \approx -0.019$, then $y \to -\infty$. Otherwise, y is asymptotic to $\sqrt{t-1}$.

21.

(a) We know that the family of solutions given by equation (19) are solutions of this initial-value problem. We want to determine if one of these passes through the point $(1,1)$. That is, we want to find $t_0 > 0$ such that if $y = [\frac{2}{3}(t-t_0)]^{3/2}$, then $(t,y) = (1,1)$. That is, we need to find $t_0 > 0$ such that $1 = \frac{2}{3}(1-t_0)$. But, the solution of this equation is $t_0 = -1/2$.

(b) From the analysis in part (a), we find a solution passing through $(2,1)$ by setting $t_0 = 1/2$.

(c) Since we need $y_0 = \pm[\frac{2}{3}(2-t_0)]^{3/2}$, we must have $|y_0| \le [\frac{4}{3}]^{3/2}$.

22.

(a) First, it is clear that $y_1(2) = -1 = y_2(2)$. Further,

$$y_1' = -1 = \frac{-t + [(t-2)^2]^{1/2}}{2} = \frac{-t + (t^2 + 4(1-t))^{1/2}}{2}$$

and

$$y_2' = -\frac{t}{2} = \frac{-t + (t^2 - t^2)^{1/2}}{2}.$$

The function y_1 is a solution for $t \ge 2$. The function y_2 is a solution for all t.

(b) Theorem 2.4.2 requires that f and $\partial f/\partial y$ be continuous in a rectangle about the point $(t_0, y_0) = (2,-1)$. Since f is not continuous if $t < 2$ and $y < -1$, the hypothesis of Theorem 2.4.2 are not satisfied.

(c) If $y = ct + c^2$, then

$$y' = c = \frac{-t + [(t+2c)^2]^{1/2}}{2} = \frac{-t + (t^2 + 4ct + 4c^2)^{1/2}}{2}.$$

Therefore, y satisfies the equation for $t \ge -2c$.

23.

(a) $\phi(t) = e^{2t} \implies \phi' = 2e^{2t}$. Therefore, $\phi' - 2\phi = 0$. Since $(c\phi)' = c\phi'$, we see that $(c\phi)' - 2c\phi = 0$. Therefore, $c\phi$ is also a solution.

(b) $\phi(t) = 1/t \implies \phi' = -1/t^2$. Therefore, $\phi' + \phi^2 = 0$. If $y = c/t$, then $y' = -c/t^2$. Therefore, $y' + y^2 = -c/t^2 + c^2/t^2 = 0$ if and only if $c^2 - c = 0$; that is, if $c = 0$ or $c = 1$.

24. If $y = \phi$ satisfies $\phi' + p(t)\phi = 0$, then $y = c\phi$ satisfies $y' + p(t)y = c\phi' + cp(t)\phi = c(\phi' + p(t)\phi) = 0$.

25. Let $y = y_1 + y_2$, then

$$y' + p(t)y = y_1' + y_2' + p(t)(y_1 + y_2) = y_1' + p(t)y_1 + y_2' + p(t)y_2 = 0.$$

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