[redacted]

1. Rewriting as $ydy = x^2dx$, then integrating both sides, we have $y^2/2 = x^3/3 + C$, or $3y^2 - 2x^3 = c; \quad y \neq 0$

2. Rewriting as $ydy = [x^2/(1 + x^3)]dx$, then integrating both sides, we have $y^2/2 = \ln|1 + x^3|/3 + C$, or $3y^2 - 2\ln|1 + x^3| = c; \quad x \neq -1, y \neq 0$

3. Rewriting as $y^{-2}dy = -\sin(x)dx$, then integrating both sides, we have $-y^{-1} = \cos(x)+C$, or $y^{-1} + \cos x = c$ if $y \neq 0$;. Also, we have $y = 0$ everywhere

4. Rewriting as $(3 + 2y)dy = (3x^2 - 1)dx$, then integrating both sides, we have $3y + y^2 - x^3 + x + C$ as long as $y \neq -3/2$.

5. Rewriting as $\sec^2(2y)dy = \cos^2(x)dx$, then integrating both sides, we have $\tan(2y)/2 = x/2 + \sin(2x)/4 + C$, or $2\tan 2y - 2x - \sin 2x = C$ as long as $\cos 2y \neq 0$. Also, if $y = \pm(2n + 1)\pi/4$ for any integer n, then $y' = 0 = \cos(2y)$

6. Rewriting as $(1 - y^2)^{-1/2}dy = dx/x$, then integrating both sides, we have $\sin^{-1}(y) = \ln|x| + C$. Therefore, $y = \sin[\ln|x| + c]$ as long as $x \neq 0$ and $|y| < 1$;. We also notice that if $y = \pm 1$, then $xy' = 0 = (1 - y^2)^{1/2}$ is a solution.

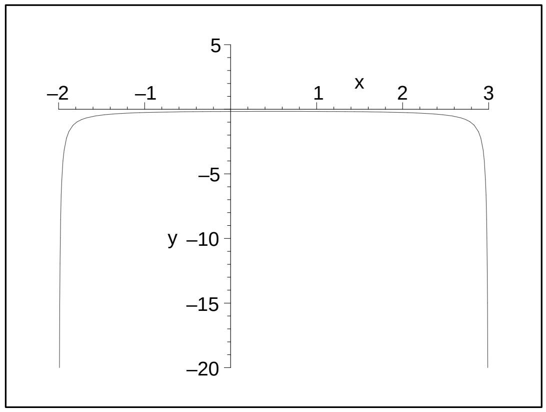
7. Rewriting as $(y + e^y)dy = (x - e^{-x})dx$, then integrating both sides, we have $y^2/2 + e^y = x^2/2 + e^{-x} + C$, or $y^2 - x^2 + 2(e^y - e^{-x}) = C$ as long as $y + e^y \neq 0$.

8. Rewriting as $(1+y^2)dy = x^2dx$, then integrating both sides, we have $y+y^3/3 = x^3/3+C$, or $3y + y^3 - x^3 = c$;.

9.

(a) Rewriting as $y^{-2}dy = (1-2x)dx$, then integrating both sides, we have $-y^{-1} = x-x^2+C$. The initial condition, $y(0) = -1/6$ implies $C = 6$. Therefore, $y = 1/(x^2 - x - 6)$.

(b)

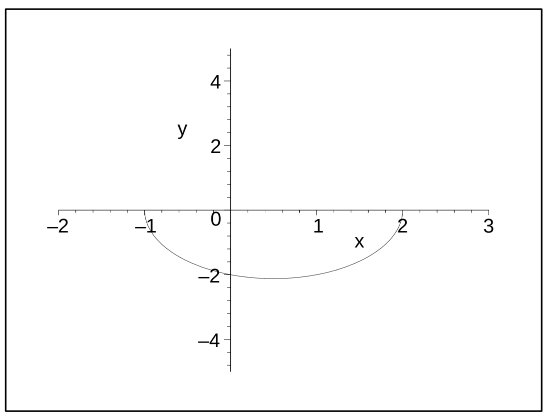


(c) $-2 < x < 3$

10.

(a) Rewriting as $ydy = (1-2x)dx$, then integrating both sides, we have $y^2/2 = x - x^2 + C$. Therefore, $y = \pm\sqrt{2x - 2x^2 + 4}$. The initial condition, $y(1) = -2$ implies $C = 2$ and $y = -\sqrt{2x - 2x^2 + 4}$.

(b)

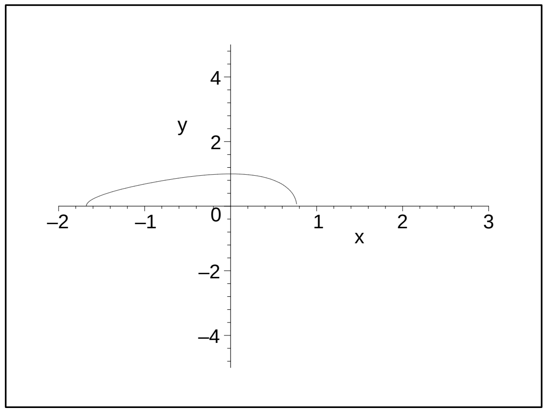


(c) $-1 < x < 2$

11.

(a) Rewriting as $xe^x dx = -ydy$, then integrating both sides, we have $xe^x - e^x = -y^2/2 + C$. The initial condition, $y(0) = 1$ implies $C = -1/2$. Therefore, $y = [2(1-x)e^x - 1]^{1/2}$.

(b)

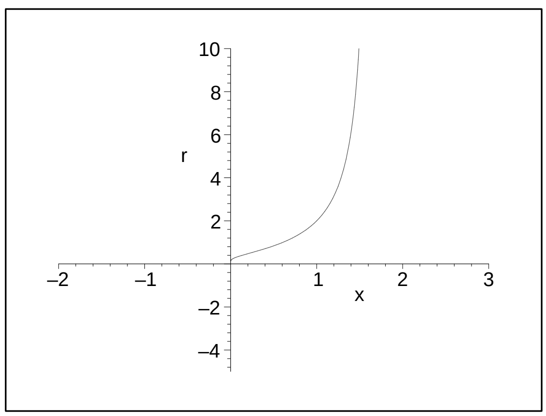


(c) $-1.68 < x < 0.77$ approximately

12.

(a) Rewriting as $r^{-2}dr = \theta^{-1}d\theta$, then integrating both sides, we have $-r^{-1} = \ln\theta + C$. The initial condition, $r(1) = 2$ implies $C = -1/2$. Therefore, $r = 2/(1 - 2\ln\theta)$.

(b)

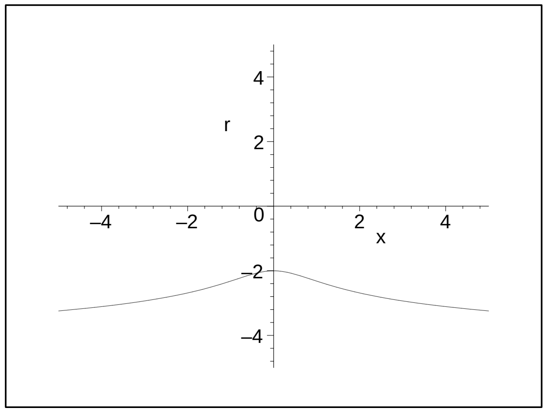


(c) $0 < \theta < \sqrt{e}$

13.

(a) Rewriting as $ydy = 2x/(1+x^2)dx$, then integrating both sides, we have $y^2/2 = \ln(1+x^2)+C$. The initial condition, $y(0) = -2$ implies $C = 2$. Therefore, $y = -[2\ln(1+x^2)+4]^{1/2}$.

(b)

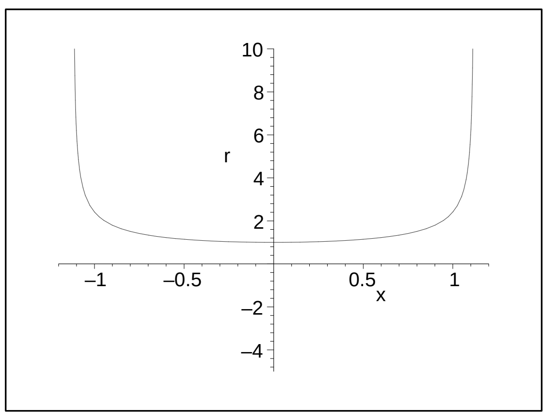


(c) $-\infty < x < \infty$

14.

(a) Rewriting as $y^{-3}dy = x(1+x^2)^{-1/2}dx$, then integrating both sides, we have $-y^{-2}/2 = \sqrt{1+x^2} + C$. The initial condition, $y(0) = 1$ implies $C = -3/2$. Therefore, $y = \left[3 - 2\sqrt{1+x^2}\right]^{-1/2}$.

(b)

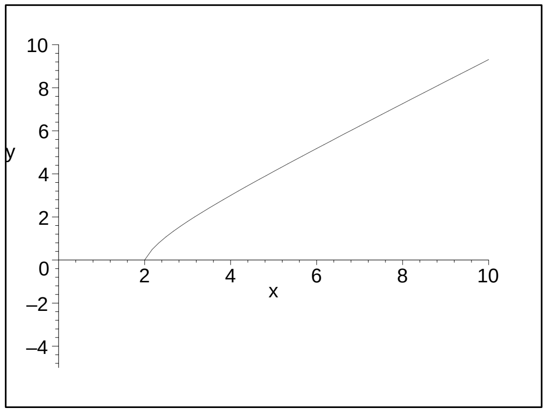


(c) $|x| < \dfrac{1}{2}\sqrt{5}$

15.

(a) Rewriting as $(1+2y)dy = 2xdx$, then integrating both sides, we have $y + y^2 = x^2 + C$. The initial condition, $y(2) = 0$ implies $C = -4$. Therefore, $y^2 + y = x^2 - 4$. Completing the square, we have $(y+1/2)^2 = x^2 - 15/4$, and, therefore, $y = -\dfrac{1}{2} + \dfrac{1}{2}\sqrt{4x^2 - 15}$.

(b)

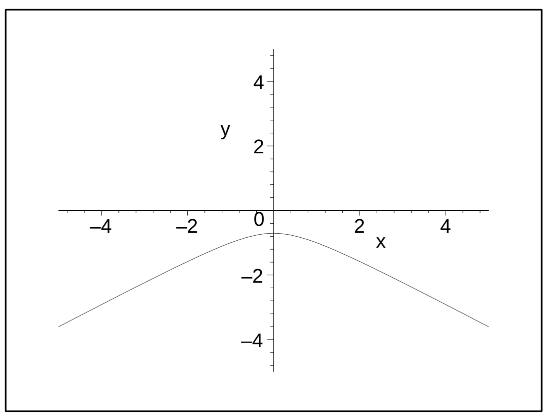


(c) $x > \frac{1}{2}\sqrt{15}$

16.

(a) Rewriting as $4y^3 dy = x(x^2+1)dx$, then integrating both sides, we have $y^4 = (x^2+1)^2/4 + C$. The initial condition, $y(0) = -1/\sqrt{2}$ implies $C = 0$. Therefore, $y = -\sqrt{(x^2+1)/2}$.

(b)

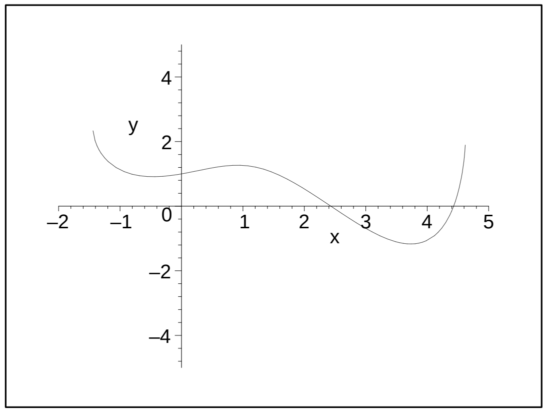


(c) $-\infty < x < \infty$

17.

(a) Rewriting as $(2y-5)dy = (3x^2 - e^x)dx$, then integrating both sides, we have $y^2 - 5y = x^3 - e^x + C$. The initial condition, $y(0) = 1$ implies $C = -3$. Completing the square, we have $(y - 5/2)^2 = x^3 - e^x + 13/4$. Therefore, $y = 5/2 - \sqrt{x^3 - e^x + 13/4}$.

(b)

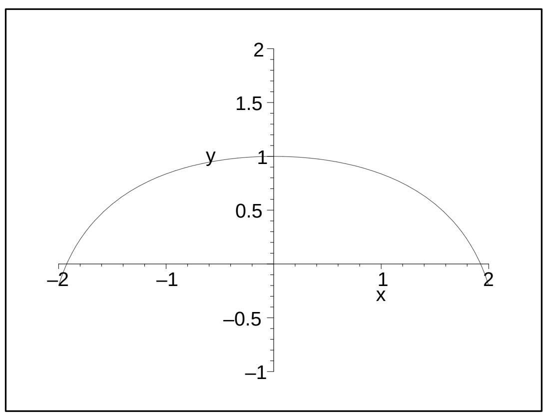


(c) $-1.4445 < x < 4.6297$ approximately

18.

(a) Rewriting as $(3+4y)dy = (e^{-x} - e^x)dx$, then integrating both sides, we have $3y + 2y^2 = -(e^x + e^{-x}) + C$. The initial condition, $y(0) = 1$ implies $C = 7$. Completing the square, we have $(y + 3/4)^2 = -(e^x + e^{-x})/2 + 65/16$. Therefore, $y = -\frac{3}{4} + \frac{1}{4}\sqrt{65 - 8e^x - 8e^{-x}}$.

(b)

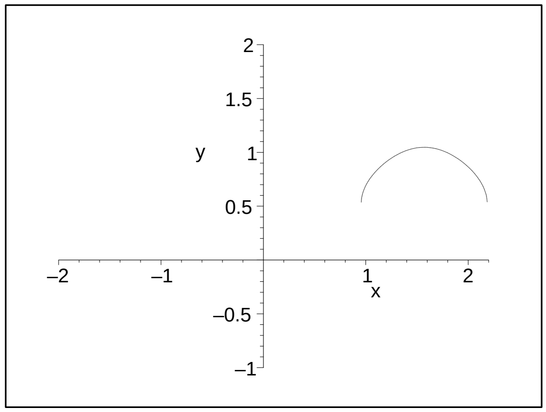


(c) $|x| < 2.0794$ approximately

19.

(a) Rewriting as $\cos(3y)dy = -\sin(2x)dx$, then integrating both sides, we have $\sin(3y)/3 = \cos(2x)/2 + C$. The initial condition, $y(\pi/2) = \pi/3$ implies $C = 1/2$. Therefore, $y = [\pi - \arcsin(3\cos^2 x)]/3$.

(b)

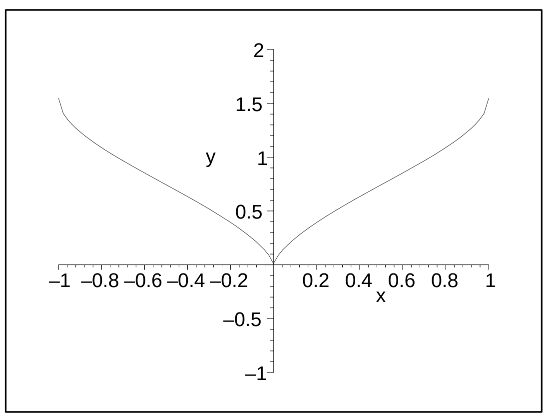


(c) $|x - \pi/2| < 0.6155$ approximately

20.

(a) Rewriting as $y^2 dy = \arcsin(x)/\sqrt{1-x^2}dx$, then integrating both sides, we have $y^3/3 = (\arcsin(x))^2/2 + C$. The initial condition, $y(0) = 1/$ implies $C = 0$. Therefore, $y = \left[\frac{3}{2}(\arcsin x)^2\right]^{1/3}$.

(b)



(c) $-1 < x < 1$

21. Rewriting the equation as $(3y^2 - 6y)dy = (1 + 3x^2)dx$ and integrating both sides, we have $y^3 - 3y^2 = x + x^3 + C$. The initial condition, $y(0) = 1$ implies $c = -2$. Therefore, $y^3 - 3y^2 - x - x^3 + 2 = 0$. When $3y^2 - 6y = 0$, the integral curve will have a vertical tangent. In particular, when $y = 0, 2$. From our solution, we see that $y = 0$ implies $x = 1$ and $y = 2$ implies $x = -1$. Therefore, the solution is defined for $-1 < x < 1$.

22. Rewriting the equation as $(3y^2 - 4)dy = 3x^2 dx$ and integrating both sides, we have $y^3 - 4y = x^3 + C$. The initial condition $y(1) = 0$ implies $C = -1$. Therefore, $y^3 - 4y - x^3 = -1$. When $3y^2 - 4 = 0$, the integral curve will have a vertical tangent. In particular, when $y = \pm 2/\sqrt{3}$. At these values for y, we have $x = -1.276, 1.598$. Therefore, the solution is defined for $-1.276 < x < 1.598$

23. Rewriting the equation as $y^{-2}dy = (2+x)dx$ and integrating both sides, we have $-y^{-1} = 2x + x^2/2 + C$. The initial condition $y(0) = 1$ implies $C = -1$. Therefore, $y = -1/(x^2/2 + 2x - 1)$. To find where the function attains it minimum value, we look where $y' = 0$. We see that $y' = 0$ implies $y = 0$ or $x = -2$. But, as seen by the solution formula, y is never zero. Further, it can be verified that $y''(-2) > 0$, and, therefore, the function attains a minimum at $x = -2$.

24. Rewriting the equation as $(3+2y)dy = (2-e^x)dx$ and integrating both sides, we have $3y + y^2 = 2x - e^x + C$. By the initial condition $y(0) = 0$, we have $C = 1$. Completing the square, it follows that $y = -3/2 + \sqrt{2x - e^x + 13/4}$. The solution is defined if $2x - e^x + 13/4 \geq 0$, that is, $-1.5 \leq x \leq 2$ (approximately). In that interval, $y = 0$ for $x = \ln 2$. It can be verified that $y''(\ln 2) < 0$, and, therefore, the function attains its maximum value at $x = \ln 2$.

25. Rewriting the equation as $(3+2y)dy = 2\cos(2x)dx$ and integrating both sides, we have $3y + y^2 = \sin(2x) + C$. By the initial condition $y(0) = -1$, we have $C = -2$. Completing the square, it follows that $y = -3/2 + \sqrt{\sin(2x) + 1/4}$. The solution is defined for $\sin(2x) + 1/4 \geq 0$. That is, $-0.126 \leq x \leq 1.44$. To find where the solution attains its maximum value, we need to check where $y' = 0$. We see that $y' = 0$ when $2\cos(2x) = 0$. In the interval of definition above, that occurs when $2x = \pi/2$, or $x = \pi/4$.

26. Rewriting this equation as $(1+y^2)^{-1}dy = 2(1+x)dx$ and integrating both sides, we have $\tan^{-1}(y) = 2x + x^2 + C$. The initial condition implies $C = 0$. Therefore, the solution is $y = \tan(x^2 + 2x)$. The solution is defined as long as $-\pi/2 < 2x + x^2 < \pi/2$. We note that $2x + x^2 \geq -1$. Further, $2x + x^2 = \pi/2$ for $x = -2.6$ and 0.6. Therefore, the solution is valid in the interval $-2.6 < x < 0.6$. We see that $y' = 0$ when $x = -1$. Furthermore, it can be verified that $y''(x) > 0$ for all x in the interval of definition. Therefore, y attains a global minimum at $x = -1$.

27.

(a) First, we rewrite the equation as $dy/[y(4-y)] = t\,dt/3$. Then, using partial fractions, we write

$$\frac{1/4}{y}\,dy + \frac{1/4}{4-y}\,dy = \frac{t}{3}\,dt.$$

Integrating both sides, we have

$$\begin{aligned} \frac{1}{4}\ln|y| - \frac{1}{4}\ln|4-y| &= \frac{t^2}{6} + C \\ \implies \ln\left|\frac{y}{y-4}\right| &= \frac{2}{3}t^2 + C \\ \implies \left|\frac{y}{y-4}\right| &= Ce^{2t^2/3}. \end{aligned}$$

From the equation, we see that $y_0 = 0 \implies C = 0 \implies y(t) = 0$ for all t. Otherwise, $y(t) > 0$ for all t or $y(t) < 0$ for all t. Therefore, if $y_0 > 0$ and $|y/(y-4)| = Ce^{2t^2/3} \to \infty$, we must have $y \to 4$. On the other hand, if $y_0 < 0$, then $y \to -\infty$ as $t \to \infty$. (In particular, $y \to -\infty$ in finite time.)

(b) For $y_0 = 0.5$, we want to find the time T when the solution first reaches the value 3.98. Using the fact that $|y/(y-4)| = Ce^{2t^2/3}$ combined with the initial condition, we have $C = 1/7$. From this equation, we now need to find T such that $|3.98/.02| = e^{2T^2/3}/7$. Solving this equation, we have $T = 3.29527$.

28.

(a) Rewriting the equation as $y^{-1}(4-y)^{-1}dy = t(1+t)^{-1}dt$ and integrating both sides, we have $\ln|y| - \ln|y-4| = 4t - 4\ln|1+t| + C$. Therefore, $|y/(y-4)| = Ce^{4t}/(1+t)^4 \to \infty$ as $t \to \infty$ which implies $y \to 4$.

(b) The initial condition $y(0) = 2$ implies $C = 1$. Therefore, $y/(y-4) = -e^{4t}/(1+t)^4$. Now we need to find T such that $3.99/-.01 = -e^{4T}/(1+T)^4$. Solving this equation, we have $T = 2.84367$.

(c) Using our results from part (b), we note that $y/(y-4) = y_0/(y_0-4)e^{4t}/(1+t)^4$. We want to find the range of initial values y_0 such that $3.99 < y < 4.01$ at time $t = 2$. Substituting $t = 2$ into the equation above, we have $y_0/(y_0-4) = (3/e^2)^4 y(2)/(y(2)-4)$. Since the function $y/(y-4)$ is monotone, we need only find the values y_0 satisfying $y_0/(y_0-4) = -399(3/e^2)^4$ and $y_0/(y_0-4) = 401(3/e^2)^4$. The solutions are $y_0 = 3.6622$ and $y_0 = 4.4042$. Therefore, we need $3.6622 < y_0 < 4.4042$.

29. We can rewrite the equation as

$$\left(\frac{cy+d}{ay+b}\right)dy = dx \implies \frac{cy}{ay+b} + \frac{d}{ay+b}dy = dx \implies \frac{c}{a} - \frac{bc}{a^2y+ab} + \frac{d}{ay+b}dy = dx.$$

Then integrating both sides, we have

$$\frac{c}{a}y - \frac{bc}{a^2}\ln|a^2y+ab| + \frac{d}{a}\ln|ay+b| = x + C.$$

Simplifying, we have

$$\begin{aligned} &\frac{c}{a}y - \frac{bc}{a^2}\ln|a| - \frac{bc}{a^2}\ln|ay+b| + \frac{d}{a}\ln|ay+b| = x + C \\ \implies &\frac{c}{a}y + \left(\frac{ad-bc}{a^2}\right)\ln|ay+b| = x + C. \end{aligned}$$

Note, in this calculation, since $\frac{bc}{a^2}\ln|a|$ is just a constant, we included it with the arbitrary constant C. This solution will exist as long as $a \neq 0$ and $ay + b \neq 0$.

30.

(a) Factoring an x out of each term in the numerator and denominator of the right-hand side, we have

$$\frac{dy}{dx} = \frac{x((y/x)-4)}{x(1-(y/x))} = \frac{(y/x)-4}{1-(y/x)},$$

as claimed.

(b) Letting $v = y/x$, we have $y = xv$, which implies that $dy/dx = v + x \cdot dv/dx$.

(c) Therefore,

$$v + x \cdot \frac{dv}{dx} = \frac{v-4}{1-v}$$

which implies that

$$x \cdot \frac{dv}{dx} = \frac{v - 4 - v(1-v)}{(1-v)} = \frac{v^2-4}{1-v}.$$

(d) To solve the equation above, we rewrite as

$$\frac{1-v}{v^2-4} dv = \frac{dx}{x}.$$

Integrating both sides of this equation, we have

$$-\frac{1}{4}\ln|v-2| - \frac{3}{4}\ln|v+2| = \ln|x| + C.$$

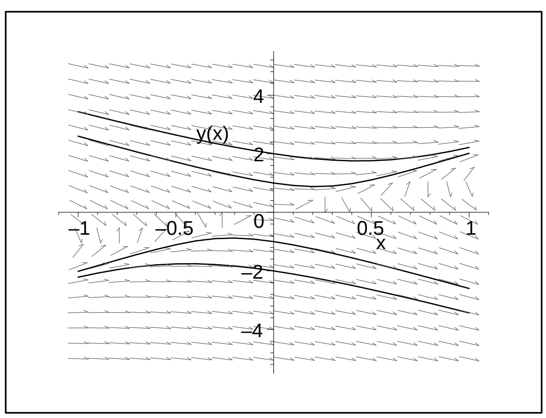
Applying the exponential function to both sides of the equation, we have

$$|v-2|^{-1/4}|v+2|^{-3/4} = C|x|.$$

(e) Replacing v with y/x, we have

$$\left|\frac{y}{x} - 2\right|^{-1/4}\left|\frac{y}{x}+2\right|^{-3/4} = C|x| \implies |x||y-2x|^{-1/4}|y+2x|^{-3/4} = C|x| \implies |y+2x|^3|y-2x| = C.$$

(f)



31.

(a)

$$\frac{dy}{dx} = 1 + (y/x) + (y/x)^2.$$

Therefore, the equation is homogeneous.

(b) The substitution $v = y/x$ results in the equation

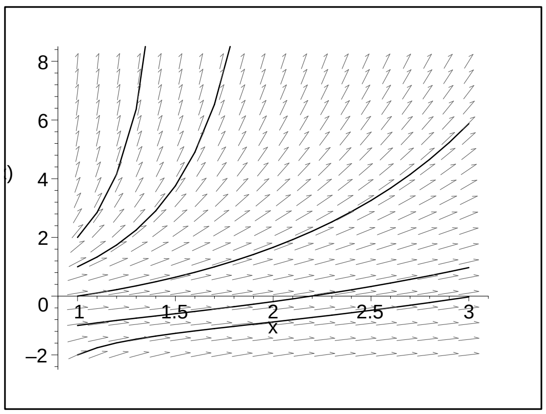
$$v + x\frac{dv}{dx} = 1 + v + v^2 \implies x\frac{dv}{dx} = 1 + v^2.$$

This equation can be rewritten as

$$\frac{dv}{1+v^2} = \frac{dx}{x}$$

which has solution $\arctan(v) = \ln|x| + c$. Rewriting back in terms of y, we have $\arctan(y/x) - \ln|x| = c$.

(c)



32.

(a)

$$\frac{dy}{dx} = (y/x)^{-1} + \frac{3}{2}(y/x).$$

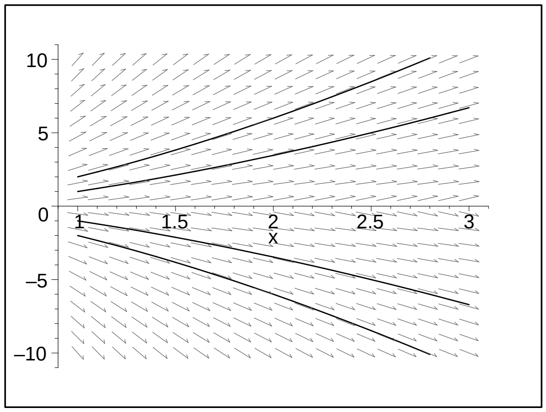
Therefore, the equation is homogeneous.

(b) The substitution $v = y/x$ results in the equation

$$v + x\frac{dv}{dx} = \frac{x^2 + 3x^2v^2}{2x^2v} \implies \frac{dv}{dx} = \frac{1+v^2}{2xv}.$$

The solution of this separable equation is $v^2 + 1 = cx$. Rewriting back in terms of y, we have $x^2 + y^2 - cx^3 = 0$.

(c)



33.

(a)

$$\frac{dy}{dx} = \frac{4(y/x) - 3}{2 - (y/x)}.$$

Therefore, the equation is homogeneous.

(b) The substitution $v = y/x$ results in the equation

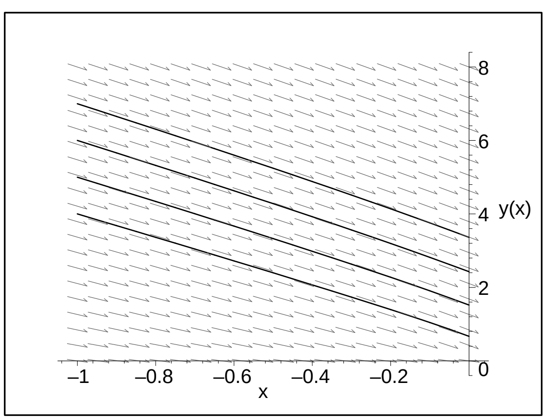
$$v + x\frac{dv}{dx} = \frac{4v - 3}{2 - v} \implies x\frac{dv}{dx} = \frac{v^2 + 2v - 3}{2 - v}.$$

This equation can be rewritten as

$$\frac{2 - v}{v^2 + 2v - 3}dv = \frac{dx}{x}.$$

Integrating both sides and simplifying, we arrive at the solution $|v + 3|^{-5/4}|v - 1|^{1/4} = |x| + c$. Rewriting back in terms of y, we have $|y - x| = c|y + 3x|^5$. We also have the solution $y = -3x$.

(c)



34.

(a)

$$\frac{dy}{dx} = -2 - \frac{y}{x}\left[2 + \frac{y}{x}\right]^{-1}.$$

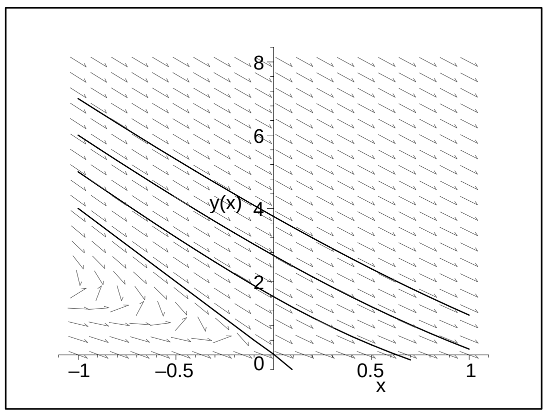
Therefore, the equation is homogeneous.

(b) The substitution $v = y/x$ results in the equation

$$v + x\frac{dv}{dx} = -2 - \frac{v}{2+v} \implies \frac{dv}{dx} = -\frac{v^2+5v+4}{x(2+v)}.$$

This equation is separable with solution $(v+4)^2|v+1| = C/x^3$. Rewriting back in terms of y, we have $|y+x|(y+4x)^2 = c$.

(c)



35.

(a)

$$\frac{dy}{dx} = \frac{1+3(y/x)}{1-(y/x)}.$$

Therefore, the equation is homogeneous.

(b) The substitution $v = y/x$ results in the equation

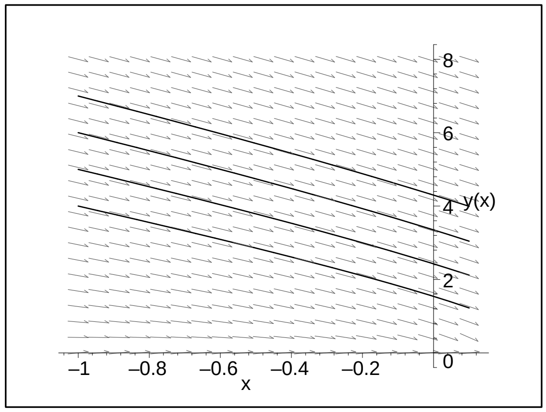
$$v + x\frac{dv}{dx} = \frac{1+3v}{1-v} \implies x\frac{dv}{dx} = \frac{v^2+2v+1}{1-v}.$$

This equation can be rewritten as

$$\frac{1-v}{v^2+2v+1}dv = \frac{dx}{x}$$

which has solution $-\frac{2}{v+1} - \ln|v+1| = \ln|x| + c$. Rewriting back in terms of y, we have $2x/(x+y) + \ln|x+y| = c$. We also have the solution $y = -x$.

(c)



36.

(a)

$$\frac{dy}{dx} = 1 + 3(y/x) + (y/x)^2.$$

Therefore, the equation is homogeneous.

(b) The substitution $v = y/x$ results in the equation

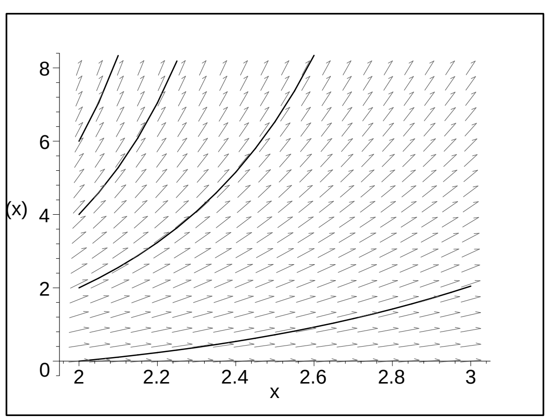
$$v + x\frac{dv}{dx} = 1 + 3v + v^2 \implies x\frac{dv}{dx} = 1 + 2v + v^2.$$

This equation can be rewritten as

$$\frac{dv}{1 + 2v + v^2} = \frac{dx}{x}$$

which has solution $-1/(v + 1) = \ln|x| + c$. Rewriting back in terms of y, we have $x/(x + y) + \ln|x| = c$. We also have the solution $y = -x$.

(c)



37.

(a)

$$\frac{dy}{dx} = \frac{1}{2}(y/x)^{-1} - \frac{3}{2}(y/x).$$

Therefore, the equation is homogeneous.

(b) The substitution $v = y/x$ results in the equation

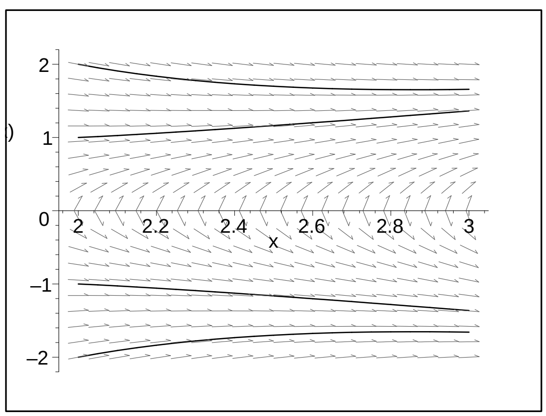
$$v + x\frac{dv}{dx} = 1 + \frac{1}{2v} - \frac{3}{2}v \implies x\frac{dv}{dx} = \frac{1-5v^2}{2v}.$$

This equation can be rewritten as

$$\frac{2v}{1-5v^2}dv = \frac{dx}{x}$$

which has solution $-\frac{1}{5}\ln|1-5v^2| = \ln|x| + c$. Applying the exponential function, we arrive at the solution $1 - 5v^2 = c/|x|^5$. Rewriting back in terms of y, we have $|x|^3(x^2 - 5y^2) = c$

(c)



38.

(a)

$$\frac{dy}{dx} = \frac{3}{2}(y/x) - \frac{1}{2}(y/x)^{-1}.$$

Therefore, the equation is homogeneous.

(b) The substitution $v = y/x$ results in the equation

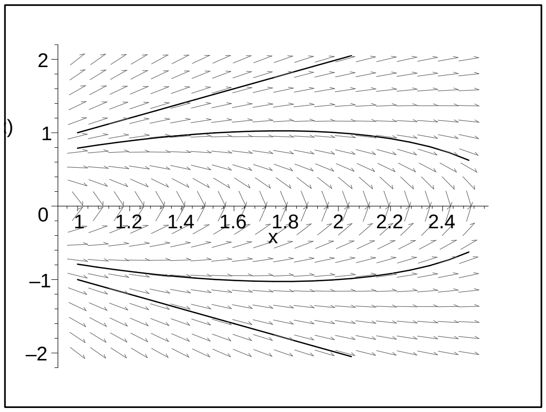
$$v + x\frac{dv}{dx} = \frac{3}{2}v - \frac{1}{2}v^{-1} \implies x\frac{dv}{dx} = \frac{v^2-1}{2v}.$$

This equation can be rewritten as

$$\frac{2v}{v^2 - 1} dv = \frac{dx}{x}$$

which has solution $\ln|v^2 - 1| = \ln|x| + c$. Applying the exponential function, we have $v^2 - 1 = C|x|$. Rewriting back in terms of y, we have $c|x|^3 = (y^2 - x^2)$

(c)



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