# Theory, development, and applications of the Advanced Compact MOSFET (ACM) model

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## ABSTRACT

This paper presents a physics-based advanced compact MOSFET (ACM) model. The ACM model is composed of very simple expressions, valid for any inversion level, conserves charge and preserves the source-drain symmetry of the transistor. We highlight the theory and approximations behind the ACM model and show its usefulness as a powerful tool for characterization, simulation, and design.

*Keywords*: charge control model, compact MOSFET model, MOS transistor

## 1 INTRODUCTION

Compact MOSFET models are indispensable tools for both circuit design and simulation. Many of the existing MOSFET models, however, are not appropriate for design due to reasons such as lack of accuracy, complex equations to describe the MOSFET behavior, excessive number of parameters, and lack of physical meaning of parameters. To some extent, these problems are a consequence of the long process of slight changes that CAD models underwent before reaching their present status.

The ACM model is the result of a fresh look at the problem of MOSFET compact modeling. It marries a physics-based approach with former semi-empirical models. ACM adopted the charge-based approach pioneered by Maher and Mead in 1987 [1] and the unified charge control model (UCCM), presented by Byun et al. in 1990 [2]. Additionally, the use of the substrate as the reference terminal allows for symmetric roles of source and drain [7, 10].

Section 2 presents the fundamental principles to derive our MOSFET model. In Section 3, we address the basic approximations that we used to deduce the unified charge control model. Next, we derive the equation of the drain current. Very simple design equations for analog circuits and the basics of parameter extraction are given in Sections 5 and 6. In Section 7, we discuss the inclusion of short-channel effects in the ACM model. Finally, a brief introduction to the computer-implemented version of the ACM model is given in Section 8.

## 2 FUNDAMENTALS

The MOSFET model hereinafter is strongly based on two physical features of the MOSFET structure: the charge-sheet model [3] and the incrementally linear relationship between the inversion charge density $Q'_I$ and the surface potential $\phi_S$ [1, 4]

$$dQ'_I = nC'_{ox}d\phi_S \quad (1a)$$
$$n = 1 + C'_b/C'_{ox} \quad (1b)$$

In (1), $C'_{ox}$ is the oxide capacitance per unit area and $C'_b$ is the depletion capacitance calculated assuming the inversion charge to be negligible. n is the slope factor, slightly dependent on the gate voltage.

In a long-channel transistor, the drain current, resulting from both the drift and diffusion transport mechanisms, is given by the charge-sheet expression [1, 5]:

$$I_D = \mu W\left(-Q'_I \frac{d\phi_S}{dx} + \phi_t \frac{dQ'_I}{dx}\right) \quad (2)$$

where $\mu$ is the carrier mobility, $\phi_t$ is the thermal voltage, W is the channel width and x is the coordinate along the channel length.

### Charge-based definitions of pinch-off and threshold voltages

The channel charge density for which the diffusion current equals the drift current will be designated the **pinch-off charge density $Q'_{ip}$**.
Substituting (1) into (2) it follows that

$$Q'_{ip} = -nC'_{ox}\phi_t \quad (3)$$

The channel-to-substrate voltage ($V_C$) for which the channel charge density equals $Q'_{ip}$ is called the **pinch-off voltage $V_P$**.

The **equilibrium threshold voltage $V_{TO}$**, measured for $V_C=0$, is the gate voltage for which the channel charge density equals $Q'_{ip}$.

The saturated drain current for a transistor with a source charge density $Q'_{ip}$ will be called **threshold current $I_{TO}$**.

## 3 THE UNIFIED CHARGE CONTROL MODEL (UCCM)

Using the gradual channel approximation, a generic cross section of the transistor can be represented by the equivalent circuit of the three terminal MOS structure shown in Fig. 1 [6].

Even though UCCM has been presented as a semi-empirical model, we have shown in [6] that UCCM can be readily derived using the charge-sheet model [3] along with two approximations:

1. the depletion capacitance per unit area is assumed to be constant along the channel and is calculated assuming the inversion charge to be negligible in the potential balance equation;

2. the inversion capacitance is proportional to the inversion charge density.

Fig. 1 shows that, for constant gate and substrate voltages, the inversion capacitance $C'_i$ is in series with the parallel connection of the oxide and depletion capacitances $(C'_{ox} + C'_b = nC'_{ox})$, leading to a simple differential equation for the charge law [6]. Using this capacitive model, the unified charge control model (UCCM) is readily derived, yielding

$$V_P - V_C = \phi_t \left[ \frac{Q'_{IP} - Q'_I}{nC'_{OX}\phi_t} + \ln\left(\frac{Q'_I}{Q'_{IP}}\right) \right] \tag{4}$$
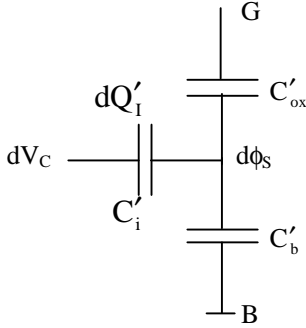


Fig. 1. Small-signal model for the three-terminal MOSFET

UCCM gives the potential as an elementary function of the inversion charge, but the charge cannot be expressed exactly as an elementary function of the potential. A useful approximation for the charge in terms of the voltage is given elsewhere [13].

As expected, in the asymptotic cases of strong and weak inversion, UCCM reduces to the conventional linear and exponential charge laws, respectively.

## 4 DRAIN CURRENT

Substituting (1) into (2) and integrating along the channel length L [8], we obtain, for the long-channel transistor,

$$I_D = I_F - I_R \tag{5a}$$

$$I_{F(R)} = \mu nC'_{ox} \frac{W}{L} \frac{\phi_t^2}{2}\left[\left(\frac{Q'_{IS(D)}}{nC'_{ox}\phi_t}\right)^2 - \frac{2Q'_{IS(D)}}{nC'_{ox}\phi_t}\right] \tag{5b}$$

where $I_{F(R)}$ is the forward (reverse) saturation current and $Q'_{IS(D)}$ is the inversion charge density evaluated at the source (drain) end. Therefore, the forward (reverse) saturation component of the current has a one-to-one relationship with the source (drain) inversion charge density. Equations (5a) and (5b) emphasize the source-drain symmetry of the MOSFET [9-10].

One can write both equations of the drain current and UCCM in terms of normalized variables. In fact, any formula in physics can be written in terms of nondimensional (normalized) variables. The choice of these variables, however, demands a clear understanding of the model as well as consistency among these variables. Indeed, these normalization variables should represent the physics behind the model. In Table I we show the fundamental parameters of the ACM model, which have a clear physical meaning.

Table I - Normalization parameters of the ACM model

| Name | Symbol | Definition |
|---|---|---|
| Thermal **voltage** | $\phi_t$ | $k_B T/q$ |
| Pinch-off **charge density** | $Q'_{ip}$ | $-nC'_{ox}\phi_t$ |
| Specific **current** (*) | $I_S$ | $\mu nC'_{ox}(\phi_t^2/2).(W/L)$ |
| Channel **length** | L | L |
| Transit **time** [5] | $\tau$ | $L^2/(2\mu\phi_t)$ |

(*) $I_S = I_{TO}/3$, where $I_{TO}$ is the drain current at threshold

## 5 EQUATIONS FOR CIRCUIT DESIGN

The following set of equations developed in [8] can be readily used for MOSFETs operating in the saturation regime:

$$\frac{\phi_t g_{ms}}{I_D} = \frac{2}{1 + \sqrt{1 + i_f}} \tag{6a}$$

$$i_f = \frac{I_D}{I_S} \tag{6b}$$

$$g_{ms} = ng_m \tag{6c}$$

$g_{ms}$ and $g_m$ are the source-transconductance and the gate-transconductance, respectively. The transconductance-to-current ratio of MOSFETs given by (6a) is a universal relationship, which is valid for any technology, dimensions and temperature. The transconductance $g_m$ depends on both the drain current $I_D$ and $i_f$, the normalized drain current [10] or inversion level. The inversion level is associated with the normalized carrier charge density at the MOSFET source [8].

The maximum voltage gain ($A_V$) of a single transistor is limited by its source-to-drain conductance $g_d$, which is approximately proportional to the ratio of the drain current to the channel length [5]. In the common-source configuration, the absolute value of the voltage gain is

$$A_V = \frac{g_m}{I_D} V_A = \frac{g_m}{I_D} V_E L \qquad (7)$$

where $V_A$ is the Early voltage and $V_E$ is the Early voltage per unit length.

The high-frequency operation of the MOSFET is limited by its internal capacitances. An indication of the transistor capability to operate at high frequencies is given by the well-known intrinsic cutoff frequency, whose value in saturation is approximated in [8] by

$$f_T \cong g_m \Big/ 2\pi(\frac{1}{2}C'_{ox}WL) = \frac{\mu\phi_t}{\pi L^2}\left(\sqrt{1+i_f}-1\right) \qquad (8)$$

where $C'_{ox}WL$ is the gate oxide capacitance. Equation (8) describes the extrapolated intrinsic cutoff frequency using the quasi-static MOSFET model. A non-quasi-static model provides a better description of the MOSFET behavior for frequencies close to the transition frequency or higher [5, 14].

Equations 6 through 8, which model relevant aspects of analog circuits, are essential to MOS amplifier design.

# 6 PARAMETER EXTRACTION

We can estimate the specific (normalization) current from the aspect ratio (W/L) and from technology parameters. To determine accurately the normalization current $I_S(V_G)$, we measure the "common-gate" characteristics in saturation ($V_D = V_G$) for several values of $V_G$, as illustrated in Fig. 2.

The slope of the curves in Fig. 2 corresponds to the logarithmic derivative of the drain current ($d\ln(I_D)/dV_S = -g_{ms}/I_D$), being a function of the forward normalized current. According to equation (6a), we can measure the values $I_{T0}$ by noting that, for $i_f = 3$ (or, equivalently, for $I_D = I_{T0}$), the logarithmic derivative of the drain current is equal to two thirds of its maximum value. The asterisks in Fig.2 depict the values of $I_{T0}$. Therefore, we can readily calculate the normalization current $I_S = I_{T0}/3$. The pinch-off voltage $V_P(V_G)$ is also extracted from the common-gate characteristics noting that, for $V_P = V_S$, the drain current is equal to $I_{T0}$. The zero-bias threshold voltage is readily determined from the $V_P(V_G)$ plot noting that $V_P(V_{T0})=0$. We emphasize here that our method to determine the fundamental MOSFET parameters is direct and independent of the secondary effects that affect the extrapolation methods based on the strong inversion characteristics of the MOSFET.
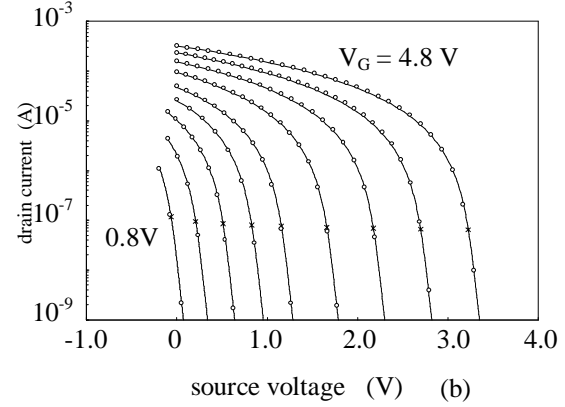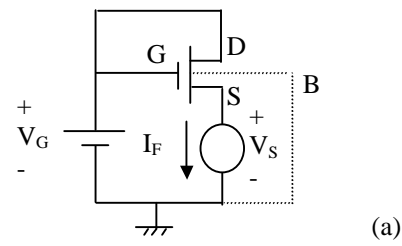


Fig. 2. (a) Circuit for measuring the common-gate characteristics in saturation. (b) Common-gate characteristics in saturation of an NMOS transistor with $t_{ox}$ = 280 Å and W = L = 25 μm ($V_G$ = 0.8, 1.2, 1.6, 2.0, 2.4, 3.0, 3.6, 4.2 and 4.8 V): (——) simulated curves calculated using the ACM model; (o) measured curves; (∗) measured points corresponding to logarithmic derivative equal to -$2/(3\phi_t)$ and $V_S = V_P$ ($i_f = 3$).

# 7 SHORT-CHANNEL EFFECTS

We have included short-channel effects in the ACM model as described in [11]. The DIBL (drain induced barrier lowering) effect is represented by parameter $\sigma$, which models the dependence of $V_P$ on both the drain and source voltages. CLM (channel length modulation) is represented by a shrinkage of the channel length $\Delta L$ given by a conventional expression [5].

We have included the effect of carrier velocity saturation in the mobility model as in [1, 11, 12]

The models of the intrinsic charges include the effects of velocity saturation, DIBL, and CLM. To calculate the total inversion charge, the channel is split into saturated and nonsaturated regions. In the saturated region, the inversion charge density is considered to be constant; therefore, the inversion charge is given by

$$Q_I = W_{eff}\int_0^{L_{eq}}Q'_I dx + W_{eff}\Delta L Q'_{IDSAT}, \qquad (9)$$

where $Q'_{IDSAT}$ is the channel charge density at saturation. dx can be written as [11]

$$dx = -\frac{\mu W_{eff}}{nC'_{ox}I_D}\left(Q'_I + Q'_{ip} - Q'_{IDSAT}\right)dQ'_I \qquad (10)$$

Using (10), the calculation of charges and capacitances is straightforward. For details, see [11, 12]. Because the ACM model uses a very simple expression for the derivative of channel charge density [6], it was the first, and is still the only model to furnish simple explicit expressions for all the intrinsic capacitive coefficients, even when short channel effects [11] are taken into account.

## 8 ACM MODEL FOR SIMULATORS

The eleven parameters of the intrinsic MOS transistor described in Table II together with W and L allow us to simulate the transistor behavior for any set of applied voltages. All parameters, excluding the last three in Table II, are conventional parameters of MOS transistors models available in SPICE-like simulators. The ACM model has been included in SMASH, a SPICE-like simulator. For more information on the implementation of the ACM model in SMASH, the reader is referred to [13].

Table II Parameters of the ACM MOSFET model

| Parameter; | Description | Unit |
|---|---|---|
| VTO | Zero-bias threshold voltage | V |
| GAMMA | Body-effect parameter | $V^{-1/2}$ |
| PHI | Surface potential | V |
| TOX | Gate oxide thickness | m |
| LD | Lateral diffusion | m |
| XJ | Junction depth | m |
| UO | Low-field mobility | $cm^2/Vs$ |
| VMAX | Saturation velocity | m/s |
| THETA | Mobility reduction parameter | $V^{-1}$ |
| SIGMA | Drain induced barrier lowering parameter | $m^2$ |
| PCLM* | Channel length modulation parameter | - |

*LAMBDA in a previous version of the ACM model

## 9 SUMMARY

The ACM model is an all-region one-equation MOS transistor model derived from fundamental concepts of physics along with proper approximations. Short-channel effects are included using a compact approach derived from physics. The ACM model is a powerful tool that can be used for hand calculations as well as computer-assisted analysis and design of MOSFET integrated circuits.

## Acknowledgements

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