Polar Volterra series with independent truncations applied in modeling of competing dual-band power amplifiers

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***Abstract*— In this article, the competing dual-band Polar Volterra series is used to model a nonlinear power amplifier (PA). The series was developed in *Matlab* software, with ten truncations separating the manipulation of the complex inputs of the model. The new series presented a reduced number of coefficients and an ability to model the PA with greater accuracy when compared to the competing dual-band polar Volterra series model of four truncations presented in a previous article.**

I. Introduction

Wireless technology is increasingly being ordered in the present time, and as the use of this technology is increasing, its modernization is necessary. In order to act productively, it is necessary to supply the high data transfer rate in the frequency spectrum without the signal interfering with the adjacent channel, avoiding distortions in the signal. The power amplifier (PA) is a device used for wireless signal amplification that must present linearity and efficiency. For the highest efficiency to be realized, the PA must act in the nonlinear region where temporal characteristics of the system in which it was applied are considered. The nonlinearity region of the PA ends up causing distortions in the signal that will be amplified and interference in adjacent channels, thus preventing the performance of these channels from being efficient. Thus, it is necessary that the PA is linearized through its behavioral modeling. The analysis of power amplifiers becomes highly complex and so simplification is required [1].

The Volterra series is a mathematical equation used with several simplifications to represent the nonlinearity of a PA, both for single band and for dual-band, through kernels and memory depth. As the polynomial order and memory depth increase, the number of coefficients generated by the series grows exponentially. The dual-band polar Volterra series separately manipulates the amplitude and phase components of the input signals with greater behavioral complexity to obtain a high accuracy of the actual PA model.

In this work, the polar Volterra Series with ten truncations will be studied, comparing it with the polar Volterra Series with four truncations [2]. The model is expected to accurately reproduce any amplifier for any frequency and bandwidth parameters. The accuracy of the model can be measured and calculated using the normalized mean square error (NMSE). Due to the increase in the number of truncations it is expected to obtain more samples and reduced NMSE values for a smaller number of coefficients, without affecting the accuracy of the model.

II. Volterra Series

Behavioral modeling will be used to reproduce the actual behaviors of an amplifier in a system. Thus, it will be possible to simulate the nonlinearity and memory effects of a PA [3]. The model uses the Volterra series described by

$$
\begin{aligned}
y_a(n) = {} & \sum_{P=1}^{P_0} \sum_{P_1=0}^{P} \cdots \sum_{P_4=0}^{P} \sum_{q_1=0}^{M} \cdots \sum_{q_{P_1}=q_{P_1-1}}^{M} \sum_{q_{P_1+1}=0}^{M} \cdots \\
& \sum_{q_{P_1+P_2}=q_{P_1+P_2-1}}^{M} \sum_{q_{P_1+P_2+1}=0}^{M} \cdots \sum_{q_{P_1+P_2+P_3}=q_{P_1+P_2+P_3-1}}^{M} \sum_{q_{P_1+P_2+P_3+1}=0}^{M} \cdots\cdots \\
& \sum_{q_r=q_{P-1}}^{M} h_{\substack{P_1,P_2,P_3,P_4,\\ q_1,q_2,\ldots,q_P}} \prod_{j_1}^{P_1} x_1(n-q_{j_1}) \qquad (1) \\
& \prod_{j_2=P_1+1}^{P_1+P_2} x_1^*(n-q_{j_2}) \prod_{j_3=P_1+P_2+1}^{P_1+P_2+P_3} x_2(n-q_{j_3}) \\
& \prod_{j_4=P_1+P_2+P_3+1}^{P_1+P_2+P_3+P_4} x_2^*(n-q_{j_4})
\end{aligned}
$$

Where P_0 is the polynomial order truncation and M is the memory length. The conjugate complexes of x_1 and x_2 are represented by x_1^* and x_2^*, $h_{P_1,P_2,P_3,P_4,q_1,q_2,\dots q_P}$ are the Kernels of the Volterra series.

For the performance of both Volterra models to be compared, simulation data of a CMOS PA circuit in class AB were used, where the sampling frequency was 120 MHz, with 3.000 samples for extraction and 2.000 for validation. Simulated in cadence specter RF software features a 2.4 GHz carrier modulated by a WiFi envelope with 20 MHz bandwidth, based on IEEE 802.11n, and the second carrier is placed at 3.5 GHz and modulated by an LTE envelope signal with a bandwidth of 20 MHz [2].

III. POLAR DUAL-BAND VOLTERRA SERIES

In the behavior modeling of dual-band PAs, the PA depends on two inputs x_1 and x_2, and two outputs y_1 and y_2. Thus, in the dual-band Volterra series, each output is a polynomial function with the memory of both inputs, considering then the interaction between the two input signals. In the polar series, the amplitude and the complex exponential of the phase are worked separately.

A. *Four Truncation polar Volterra series*

In the Polar Volterra series of competing dual-band of four truncation dual-band series approached by (1). the complex band 1 output at present time, y_1 (n), is calculated from:

$$
\begin{aligned}
&\sum_{p_1=1}^{P_1}\sum_{r=1}^{p_1}\sum_{q_1=0}^{M_1}\cdots\sum_{q_r=q_r-1}^{M_1}\sum_{q_{r+1}=0}^{M_1} \\
&\cdots\sum_{q_{p_1}=q_{p_1-1}}^{M_1}\ \sum_{p_2=1}^{P_2}\sum_{s=1}^{p_2}\sum_{l_1=0}^{M_2}\cdots\sum_{l_s=l_{s-1}}^{M_2}\sum_{l_s=l_{s+1}}^{M_2} \\
&\cdots\sum_{l_{2s-1}=l_{2s-2}}^{M_2}\ \sum_{l_{2s}=0}^{M_2}\cdots\sum_{l_{s+p_2-1}=l_{s+p_2-2}}^{M_2}\ \sum_{l_{s+p_2}=0}^{M} \\
&\cdots\sum_{l_{2p_2-1}=l_{2p_2-2}}^{M_2} h_{p_1,r,q_1,\dots,q_{p_1},p_2,s,l_1,\dots,l_{2p_2-1}} \\
&\prod_{j_1=1}^{r} a_1(n-q_{j_1})\prod_{j_2=r+1}^{p_1} a_2(n-q_{j_2}) \qquad (2) \\
&\prod_{k_1=1}^{s}\exp(j\theta_1(n-l_{k_1}))\prod_{k_2=s+1}^{2_s-1}\exp(-j\theta_1(n-l_{k_2})) \\
&\prod_{k_3=1}^{p_2-s}\exp(j\theta_2(n-l_{k_3}))\prod_{k_4=1}^{p_2-s}\exp(-j\theta_2(n-l_{k_4}))
\end{aligned}
$$

Where a_1 and a_2 are the respective amplitudes of inputs x_1 and x_2, θ_1 and θ_2 are the respective phase components of inputs x_1 and x_2.The positive number of phases of θ_1 is equal to the negative number of phases plus one, while the positive number of phases of θ_2 is equal to the negative number of phases of θ_2. In this four truncations model, P_1 and M_1 were associated with amplitudes a_1 and a_2 and P_2 and M_2 were associated with phases θ_1 and θ_2.

The four truncations polar Volterra series despite obtaining adequate NMSE values for both WiFi and LTE bands with P_1=5, M_1 =1, P_2=1 and M_2=1, showed a super fit when trying to represent memory effects, with values below of -40 dB during the parameter extraction and greater than -18 dB for data not used in the extraction [2].

B. *Ten truncation polar Volterra series*

In this article, the ten truncation dual-band polar Volterra series is developed from the four truncation dual-band polar Volterra series. Replicating the series (2) four times, adjusting the values of truncation factors in its minimum values and through simplifications, in this new series the complex band 1 output at present time, y_1 (n), is calculated from:

$$
\begin{aligned}
&\sum_{p_1=1}^{P_{11}}\sum_{r=1}^{p_1} h_{p_1,r}(0).a_1^{r}(n).a_2^{p_1-r}(n).\exp(j\theta_1(n))+ \\
&\sum_{p_1=1}^{P_{12}}\sum_{r=1}^{p_1}\sum_{q_1=0}^{M_2}\cdots\sum_{q_r=q_r-1}^{M_2}\sum_{q_r+1=0}^{M_2}\cdots\cdot \\
&\sum_{q_{p_1}=q_{p_1}-1}^{M_2} h_{p_1,r,q_1,\dots,q_{p_1}}\prod_{j_1=1}^{r} a_1(n-q_{j_1}) \\
&\prod_{j_2=r+1}^{p_1} a_2(n-q_{j_2})\exp(j\theta_1(n))+ \\
&\sum_{p_1=1}^{P_{13}}\sum_{r=1}^{p_1}\sum_{q_1=0}^{M_3}\cdots\sum_{q_r=q_{r-1}}^{M_3}\sum_{l_1=0}^{L_3}\sum_{q_r+1=0}^{M_3}\cdots \\
&\sum_{q_{p_1}=q_{p_1}-1}^{M_3} h_{p_1,r,,q_1,\dots,q_{p_1},p_2,s,l_1}\prod_{j_1=1}^{r} a_1(n-q_{j_1}) \\
&\prod_{j_2=r+1}^{p_1} a_2(n-q_{j_2})\exp(j\theta_1(n-l_1))+ \qquad (3) \\
&\sum_{p_1=1}^{P_{14}}\sum_{r=1}^{p_1}\sum_{q_1=0}^{M_4}\cdots\sum_{q_r=q_r-1}^{M_4}\sum_{q_{r+1}=0}^{M_4}\cdots\sum_{q_{p_1}=q_{p_1-1}}^{M_4}\ \sum_{p_2=1}^{P_{24}}\sum_{s=1}^{p_2}\sum_{l_1=0}^{L_4}\cdots \\
&\sum_{l_s=l_{s-1}}^{L_4}\ \sum_{l_{s+1}=0}^{L_4}\cdots\sum_{l_{2s-1}=l_{2s-2}}^{L_4}\ \sum_{l_{2s}=0}^{L_4}\cdots \\
&\quad\sum_{l_{s+p_2-1}=l_{s+p_2-2}}^{L_4}\ \sum_{l_{s+p_2}=0}^{L_4}\cdots\sum_{l_{2p_2=1}=l_{2p_2-2}}^{L_4} h_{p_1,r,q_1,\dots,q_{p_1},p_2,l_1,\dots,l_{2p_2-1}} \\
&\prod_{j_1=1}^{r} a_1(n-q_{j_1})\prod_{j_2=r+1}^{p_1} a_2(n-q_{j_2}) \\
&\prod_{K_1=1}^{s}\exp(j\theta_1(n-l_{k_1}))\prod_{K_2=s+1}^{2s-1}\exp(-j\theta_1(n-l_{k_2})) \\
&\prod_{K_3=1}^{p_2-s}\exp(j\theta_2(n-l_{k_3}))\prod_{K_4=1}^{p_2-s}\exp(-j\theta_2(n-l_{k_4}))
\end{aligned}
$$

Similar to the terms of (2), a_1 and a_2 are the respective amplitudes of inputs x_1 and x_2, θ_1 and θ_2 are the respective phase components of inputs x_1 and x_2. The positive number of phases of θ_1 is equal to the negative number of phases plus one, while the positive number of phases of θ_2 is equal to the negative number of phases of θ_2. In this ten truncations model, P_{11}, P_{12}, P_{13}, P_{14} and M_2, M_3, M_4 were associated with amplitudes a_1 and a_2 and P_{24} and L_3, L_4 was associated with phases θ_1 and θ_2.

As the number of truncations in the series increases, there is a greater amount of polynomials and memory duration. Thus, the Volterra series generates many coefficients that make the computational effort very large. To reduce this number of coefficients and maintain the accuracy of the model, the complex inputs of the amplifier are manipulated. This manipulation consists of determining independent truncation values, thus obtaining four divisions in the model, with six truncations adjusted. Truncation values can be selected independently, but the evaluation of each candidate is time-consuming. Therefore, to reduce the number of candidates to be evaluated the values are adjusted in their minimum values established by the following conditions: for polynomials that model the nonlinearity of amplitude $P_{11} >= P_{12} >= P_{13} >= P_{14}$, for memories that model amplitude memory $M_2 >= M_3 >= M_4$, and for memories that model phase memory $L_3 >= L_4$ [4]. The new model has ten truncation factors to be determined, and conditions must be met to reduce the number of parameters. For each subdivision, nonlinearity functions of amplitude, phase linearity, without or with amplitude and phase memories were assumed and phase nonlinearity.

The first division does not reproduce the effects with memory, the output being a function of the current input. A truncation factor is determined, which is the polynomial order of amplitude P_{11}.

The second division has two truncation factors to be determined, the polynomial order of amplitude P_{12} and the memory duration of the M_2 amplitude and is capable of reproducing memory effects related to previous inputs.

The third division has three truncation factors to be determined, the polynomial order of amplitude P_{13}, the length of amplitude memory M_3 and the length of phase memory L_3. It can consider both amplitude and phase memory effects.

The fourth division has four truncation factors to be determined, the polynomial order of amplitude P_{14}, the memory length of amplitude M_4, the polynomial order of phase P_{24} and the length of the L_4 phase memory. The complex instantaneous output is a non-linear function of the input amplitudes and phases of instantaneous and previous inputs.

IV. Simulation Results

The results were obtained in Matlab software using double precision floating point arithmetic and using the modeling data described in Section II. Comparing the polar Volterra series of 10 truncations with the Volterra series of 4 truncations, it was possible to verify the efficiency of this new series using many more truncation combinations. The old model contains four truncations, were P_1 ranging from 1 to 5, M_1 from 0 to 2, P_2 from 1 to 3 and M_2 from 0 to 2. The new model contains 10 truncations. The amplitudes and phases vary as follows P_{11}, P_{12}, P_{13}, P_{14} and P_{24} vary from 1 to 5 and M_2, M_3, M_4, L_3 and L_4 vary from 0 to 2.

On average, the NMSE value for the four truncation model was -24.16 dB for the WiFi band and -29.07 dB for the LTE band, for the new model the average NMSE value was -32.05 dB for the WiFi band and -31.70 dB for the LTE band. The worst values obtained for each model of the WiFi band output were 33.21 dB for the four truncations and -20.68 dB for the new model. For the LTE band, the worst results were -17.62 dB for the four truncation model and -27.40 dB for the new model. In Table 1 it is possible to observe the best NMSE values obtained for both models.

TABLE 1. BEST NMSE VALUES

Models	WiFi (dB)	LTE (dB)
New model	-40.37	-39.70
Old model	-37.61	-37.54

From Table 1, in the best NMSE values were observed improvements of 2.8 dB for the WiFi band and 2.2 dB for the LTE band. For the old model, the best result was obtained with P_1= 5, M_1= 1, P_2= 1 and M_2 = 1 for the WiFi and LTE bands, considering the memory effects in the system. For both bands a higher value was adopted for the polynomial order of amplitude, while the same value is used for amplitude and phase memory lengths. For the new model, the best result was obtained with P_{11}=5, P_{12}=2, P_{13}=1, P_{14}=1, P_{24}=1, M_2=2, M_3=2, M_4=2, L_3=2, L_4=2, for the WiFi band and the best result was obtained with P_{11}=5, P_{12}=2, P_{13}=1, P_{14}=1, P_{24}=1, M_2=2, M_3=0, M_4=0, L_3=1, L_4=1, for the LTE band.

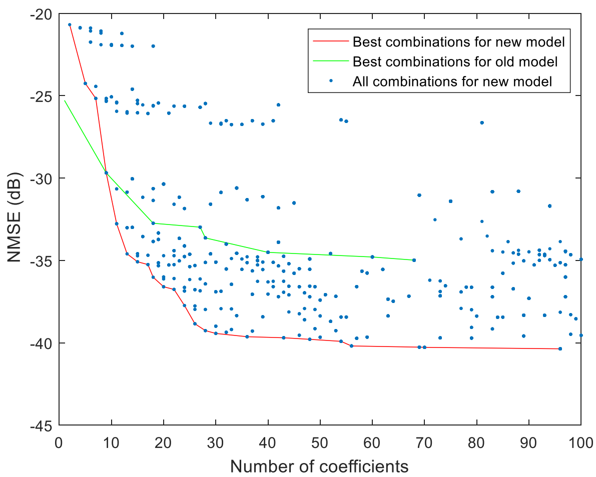

Fig. 1. NMSE values as a function of the number of coefficients for the WiFi band.

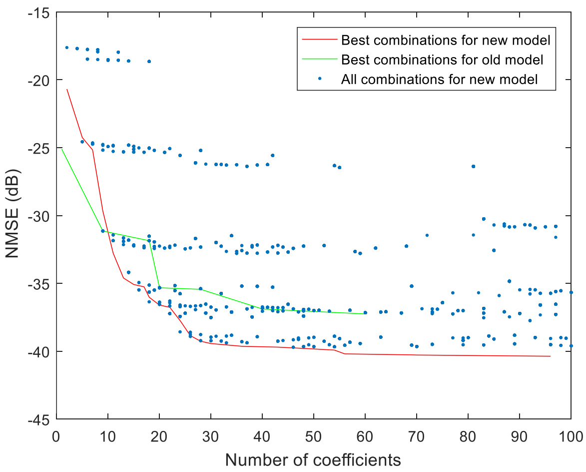

Fig. 2. NMSE values as a function of the number of coefficients for the LTE band.

In Figures 1 and 2 it is possible to affirm that even with the reduction in the number of coefficients generated by the new series, there was no deterioration in the modeling accuracy. The new model presented a more accurate and efficient curve in the range of NMSE values with acceptable accuracies, above ten coefficients, than the curve obtained for the old model, so the new model was able to represent the real amplifier more efficiently.

V. Conclusions

The polar Volterra polar series of dual-band competitor of ten independent truncations presents a complexity similar to the polar series of four truncations. Because the series works only with input and output data from the amplifier, it is expected that any amplifier can be modeled accurately. The extraction of the amplitude and exponential complex of the phase was performed to be able to work with them in isolation both in the new series and in the previous one. From distinct behaviors and ten truncations previously defined in subdivisions for polynomials and memories, the new series was able to generate more combinations of truncations with more accurate coefficients and NMSEs for both bands, being the best values for the WiFi band and the LTE band of -40.37 dB and -39.70 dB respectively, more efficiently modeling the PA.

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